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TURBULENCE VISCOSITY IN VERTICAL ADIABATIC GAS–LIQUID FLOW

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Abstract—An empirical correlation for the turbulence viscosity in two-phase flow is developed, based on the assumption that the fluctuations of the turbulent velocity may be divided into two components: one due to the momentum exchange of the liquid phase, the other due to the movement of the dispersed phase. The reliability of the correlation is checked against measurements from various sources, showing a standard deviation of 22 per cent.

1. INTRODUCTION

In single phase flow it is customary to express the behavior of the turbulence stresses in the equations of motion in terms of the mean-velocity gradients, based on the assumption that the turbulence stresses act like the viscous stresses. This eddy or turbulence viscosity concept was introduced by Boussinesq, who considered the turbulence stresses directly proportional to the velocity gradient (e.g. Hinze 1975). A correlation for the turbulence viscosity ϵ_m in channel flow is known from measurements. However, to the author's knowledge, no correlations exist for the turbulence viscosity in two-phase gas–liquid flows. Such a correlation would be useful as it would permit to calculate the viscosity terms in the momentum balances in case of two dimensional computations for gas–liquid flows. The present paper tries to establish such a correlation from available experimental data on two-phase flow pressure loss. For this purpose an analogy in certain aspects is assumed between single phase and two phase flow. At first a method is outlined to obtain an expression for the frictional pressure drop in single phase flow as a function of the turbulence viscosity and the velocity gradient at the wall. The method is based on the fact that ϵ_m is almost constant in the core region of pipe flow and that the velocity distribution can be approximated by a power law.

Subsequently the same method will be applied to one-dimensional two-phase flow, on the assumption that two analogies between this type of flow and single phase flow are valid, viz. that the turbulence viscosity is constant in the core region of the channel and that the velocity distributions are similar. It is further assumed that the turbulence stresses may be subdivided into a part due to the movement of the liquid phase and another part attributed to the momentum exchange caused by the presence or relative movement of the gas phase. This approach is used by Sato & Sekoguchi (1975) for predicting the liquid velocity distribution in two phase bubble flow. Thus an expression is obtained for the frictional pressure loss in two phase flow, again as a function of the turbulence viscosity and the liquid velocity gradient, which in turn is used in reversed sense, i.e. to compute the turbulence viscosity from available experimental data on two-phase frictional pressure loss.

2. PRESSURE DROP IN SINGLE PHASE FLOW

The momentum balance equation for steady, homogeneous, axisymmetric flow through a pipe of constant circular cross-section yields (e.g. Hinze 1975):

$$\frac{\rho}{r}\frac{\partial}{\partial r}(ru_r u_z) = -\frac{\partial p}{\partial z} + \eta\nabla^2 u_z \qquad [1]$$

where r and z denote the radial and axial coordinate direction, respectively, and the operator ∇^2 is defined by

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{\partial}{r\partial z}.$$

The momentum equation for turbulent flow is obtained by applying the well-known Reynolds' procedure to [1] by substituting

$$u = \bar{u} + u'$$

and

$$p = \bar{p} + p'$$

where the overbar denotes time-averaged values and the prime denotes the fluctuating components.

Averaging with respect to time thus yields

$$\frac{\rho}{r}\frac{\partial}{\partial r}(r\bar{u}_r\bar{u}_z) + \frac{\rho}{r}\frac{\partial}{\partial r}(r\overline{u'_r u'_z}) = -\frac{\partial p}{\partial z} + \eta\left(\frac{\partial^2 \bar{u}_z}{\partial r^2} + \frac{1}{r}\frac{\partial \bar{u}_z}{\partial r}\right). \quad [2]$$

For fully developed and axi-symmetric flow $\bar{u}_r = 0$, hence [2] reduces to

$$\frac{\mathrm{d}\bar{p}}{\mathrm{d}z} = \frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}r\left(\eta\frac{\mathrm{d}u_z}{\mathrm{d}r} - \rho\overline{u'_r u'_z}\right) \quad [3]$$

or

$$\frac{\mathrm{d}\bar{p}}{\mathrm{d}z} = \frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}(r\tau_{rz}) \quad [4]$$

where τ_{rz}, the sum of the molecular and turbulent shear stresses, is now defined by

$$\tau_{rz} = \eta\frac{\mathrm{d}\bar{u}_z}{\mathrm{d}r} - \rho\overline{u'_r u'_z}. \quad [5]$$

The term $\rho\overline{u'_r u'_z}$, called the turbulence or Reynolds' stress, is assumed to be directly proportional to the velocity gradient, according to the hypothesis of Boussinesq, yielding

$$\tau_{rz} = (\eta + \rho\epsilon_m)\frac{\mathrm{d}\bar{u}_z}{\mathrm{d}r} \quad [6]$$

where ϵ_m is the so-called eddy or turbulence viscosity. It is known that the shear stress distribution is linear (e.g. Hinze 1975). The frictional pressure drop is obtained after integration of [4] over the entire cross section of the pipe, yielding

$$\left(\frac{\mathrm{d}p}{\mathrm{d}z}\right)_{fr} = \frac{4}{D}\tau_w \quad [7]$$

where τ_w is the wall shear stress, which can be determined from [6] if the turbulence viscosity and the velocity gradient at the wall are known.

● Measurements by Nikuradse and Nunner have shown that the velocity distribution can be approximated by a power law

$$u_z = u_{z\max}\left(1-\frac{2r}{D}\right)^{1/n}. \quad [8]$$

According to Nunner (e.g. Hinze 1975) the value for n is found from

$$\frac{1}{n} = \sqrt{\lambda} \quad [9]$$

where the friction factor λ is defined by

$$\left(\frac{\mathrm{d}p}{\mathrm{d}z}\right)_{fr} = \frac{\lambda}{D}\frac{1}{2}\rho\langle u_z\rangle^2 \dagger \quad [10]$$

● Regarding the turbulence viscosity it is known from experiments by Laufer and Nunner (see Hinze 1975) that ϵ_m is almost constant in the core region of pipe flow, yielding

$$\frac{\epsilon_m}{\nu} = \frac{\mathrm{Re}_D}{30}\sqrt{\frac{\lambda}{8}} \quad [11]$$

where Re_D is defined by

$$\mathrm{Re}_D = \frac{\rho\langle u_z\rangle D}{\eta} \quad [12]$$

and the friction factor λ is given by the correlation for smooth pipes in the Moody diagram (cf. Drew *et al.* 1932):

$$\lambda = 0.0056 + \frac{0.5}{\mathrm{Re}_D^{0.32}}. \quad [13]$$

However, as neither the velocity distribution given in [8] nor the assumption of uniform turbulence viscosity are valid in the wall region, it is not easy to satisfy the condition that the linear shear stress distribution yields the actual shear stress at the wall. Assumption of a slightly modified distribution for the shear stress (see figure 1) makes it possible to find the radial position where the velocity gradient is such that the shear stress determined from [6] equals the wall shear stress τ_w. Thus the following relation should be satisfied

$$\eta\left(1+\frac{\epsilon_m}{\nu}\right)\left(\frac{\mathrm{d}u_z}{\mathrm{d}r}\right)_r = \tau_w \quad [14]$$

where the subscript r refers to the radial position to be found. To evaluate relation [11] we shall first have to establish expressions for $(\mathrm{d}u_z/\mathrm{d}r)$ and τ_w.

● The velocity gradient is obtained by differentiating [8]. As the cross-sectional averaged value $\langle u_z\rangle$ is known instead of $u_{z\max}$, [8] is integrated over the cross section and divided by the area $(\pi/4)D^2$, yielding

$$\langle u_z\rangle = u_{z\max}\frac{8}{D^2}\int_0^{D/2}\left(1-\frac{2r}{D}\right)^{\sqrt{\lambda}} r\,\mathrm{d}r$$

†The bar over u_z will be omitted for convenience.

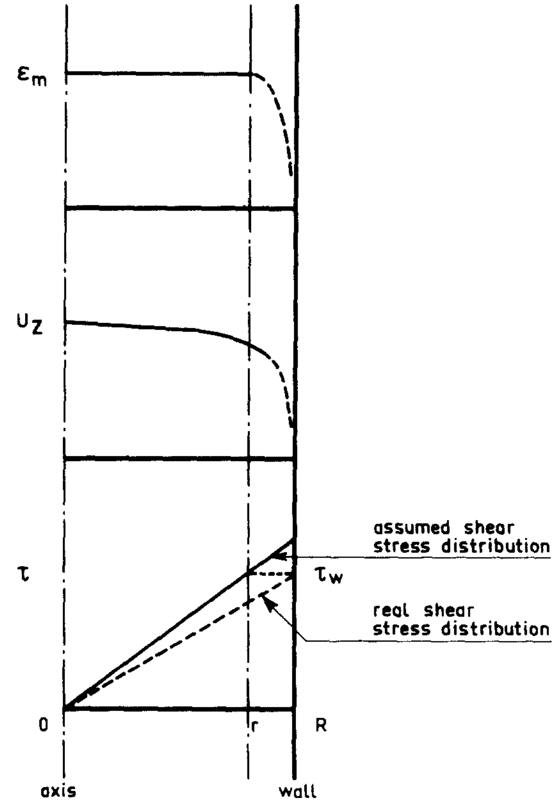

Figure 1. Distributions for turbulence viscosity, velocity and shear stress.

or

$$\langle u_z \rangle = \frac{2u_{z\max}}{(\sqrt{\lambda}+1)(\sqrt{\lambda}+2)}. \qquad [15]$$

This equation is substituted in [8], yielding

$$u_z = \langle u_z \rangle \frac{1}{2}(\sqrt{\lambda}+2)\left(1-\frac{2r}{D}\right)^{\sqrt{\lambda}}.$$

Differentiation with respect to r gives

$$\frac{\mathrm{d}u_z}{\mathrm{d}r} = \frac{\langle u_z \rangle}{D}\sqrt{\lambda}(\sqrt{\lambda}+1)(\sqrt{\lambda}+2)\left(1-\frac{2r}{D}\right)^{\sqrt{(\lambda-1)}}. \qquad [16]$$

● The wall shear stress τ_w is obtained from [7], where it is known that the frictional pressure drop can also be related to the momentum flux of the fluid, according to [10], yielding

$$\frac{\lambda}{D}\frac{1}{2}\rho\langle u_z \rangle^2 = \frac{4}{D}\tau_w$$

or

$$\tau_w = \frac{\lambda}{8}\rho\langle u_z \rangle^2. \qquad [17]$$

● With [15]–[17] it is now possible to satisfy [14]. Substitution of [16] and [17] into [14] yields

$$\eta\left(1+\frac{\epsilon_m}{\nu}\right)\frac{\langle u_z\rangle}{D}a\left(1-\frac{2r}{D}\right)^{\sqrt{\lambda}-1}=\frac{\lambda}{8}\rho\langle u_z\rangle^2 \qquad [18]$$

where

$$a=\sqrt{\lambda}(\sqrt{\lambda}+1)(\sqrt{\lambda}+2). \qquad [19]$$

After some rearrangement, [18] yields

$$\left(1-\frac{2r}{D}\right)^{\sqrt{\lambda}-1}=\frac{\lambda \mathrm{Re}_D}{8a\left(1+\frac{\epsilon_m}{\nu}\right)}. \qquad [20]$$

With the aid of [11], [20] reads:

$$\frac{2r}{D}=1-\left(\frac{\lambda \mathrm{Re}_D}{8a\left(1+\frac{\mathrm{Re}_D}{30}\sqrt{\frac{\lambda}{8}}\right)}\right)^{(1/\sqrt{(\lambda-1)})} \qquad [21]$$

where λ is given by [13] and the factor a by [19]. The following table shows $(2r/D)$ as a function of the number Re_D.

Re_D	$2r/D$
10^4	0.812
10^5	0.817
10^6	0.816
10^7	0.816
10^8	0.815

It is clear that for the range of practical interest a value for $(2r/D)$ of 0.816 is a good approximation. This may be illustrated with the aid of a subset of 16 measurements on frictional pressure loss in single phase water flow, taken from the total set used in subsection 4.1. For these measurements, obtained by Malnes (1966) and Niese (1968), the frictional pressure drop was computed using [6] and [7], in which $(\mathrm{d}u_z/\mathrm{d}r)$ is obtained from [16] and $2r/D = 0.816$. Figure 2 confirms that the computed and measured values of the frictional pressure loss are in good agreement. A second comparison was made with the aid of two well-known statistical parameters, viz. the arithmetic mean deviation (M.D.) $\bar{\delta}$ and the standard deviation (S.D.) σ, defined in the way proposed by Dukler *et al.* (1964). The fractional deviation is given by

$$\delta_i=\frac{P_i-M_i}{M_i}\cdot 100 \text{ per cent} \qquad [22]$$

where P_i and M_i are the predicted and measured values for the ith measurement.

Hence the M.D. is

$$\bar{\delta}=\frac{1}{n}\sum_{i=1}^{n}\delta_i \qquad [23]$$

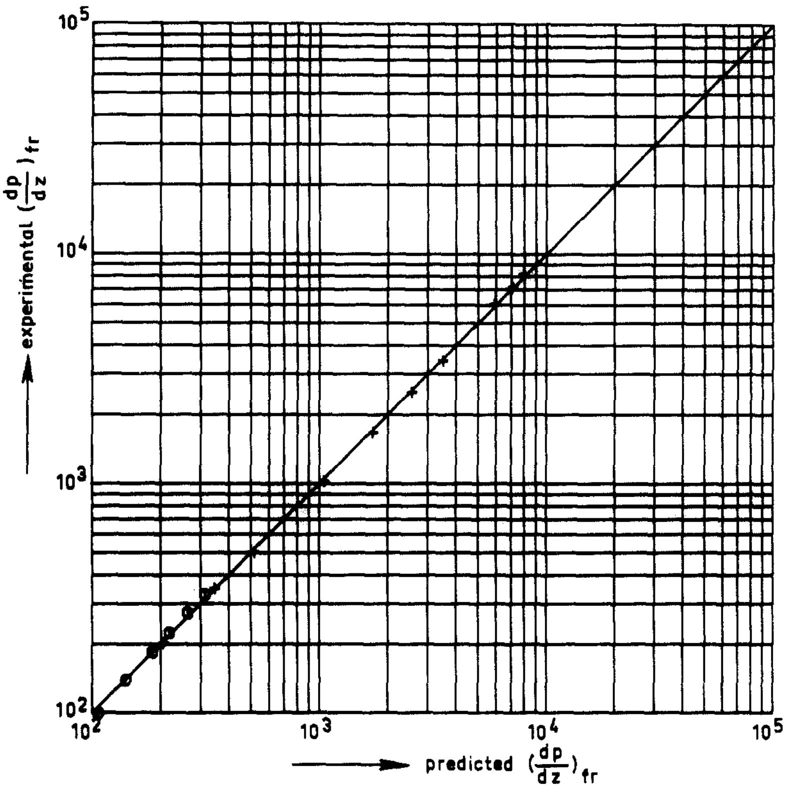

Figure 2. Computed and measured frictional pressure loss for water flow.

and the S.D.

$$\sigma = \sqrt{\left(\sum_{i=1}^{n} (\delta_i - \bar{\delta})^2\right)/(n-1)} \qquad [24]$$

where n is the number of measurements.

The computed frictional pressure loss values approximate the measured values within a M.D. of 0.02 per cent and a S.D. of 3.22 per cent.

It is worthwhile to note that the position $2r/D = 0.816$ is close enough to the wall to validate the assumed shear stress distribution shown in figure 1.

3. ANALYSIS FOR TWO-PHASE FLOW

In accordance with Sato & Sekoguchi (1975) we now assume the averaging procedure of Reynolds, applied in section 2 for single-phase flow, to be also applicable to two-phase flows, where the liquid phase is incompressible and the gas phase behaves only as a voidage. Hence only the shear stress in the liquid phase will be considered. It is further assumed that the turbulent fluctuations can be divided into two components, caused by the movement of the liquid and by phase interaction, respectively. Thus

$$u = \bar{u} + (u')_m + (u')_i$$

where the indices denote (liquid) momentum and phase interaction, respectively, leading to the following expression for the shear stress in two-phase flow:

$$\tau_{rz} = (1-\alpha)\left(\eta \frac{d\bar{u}_{Lz}}{dr} - \rho_L(\overline{u'_r u'_z})_m - \rho_L(\overline{u'_r u'_z})_i\right). \qquad [25]$$

With the hypothesis of Boussinesq:

$$\tau_{rz} = (1-\alpha)\eta_L\left(1+\frac{\epsilon_m}{\nu_L}+\frac{\epsilon_i}{\nu_L}\right)\frac{\mathrm{d}u_{Lz}}{\mathrm{d}r} \qquad [26]$$

where ϵ_i represents the turbulent viscosity due to phase interaction. Equation [7] is still valid, yielding

$$\left(\frac{\mathrm{d}p}{\mathrm{d}z}\right)_{fr} = \frac{4}{D}\tau_w$$

where τ_w is obtained from [26] using the same procedure as outlined in the foregoing section. For this purpose the following assumptions must be made:

—The liquid velocity distribution is similar to that in single phase flow, as was found by Sato *et al.* (1975) and Serizawa *et al.* (1975), and may hence be approximated by the power law of [8].

—The velocity gradient [16] for which $\tau_{rz} = \tau_w$ occurs at the same radial position as was found for single phase flow, i.e. $2r/D = 0.816$.

—Both ϵ_m and ϵ_i are almost constant in the core region of the pipe, as validated by Sato *et al.* (1975).

Correlations for ϵ_m and λ are obtained by slightly modifying the single phase expressions [11] and [13]. Hence

$$\frac{\epsilon_m}{\nu_L} = \frac{\mathrm{Re}_{Dtp}}{30}\sqrt{\left(\frac{\lambda_{tp}}{8}\right)} \qquad [27]$$

and in the same sense

$$\lambda_{tp} = 0.0056 + \frac{0.5}{\mathrm{Re}_{Dtp}^{0.32}}. \qquad [28]$$

In the literature opinions on the proper difinition of Re_{tp} appear to vary. The present author has chosen the following approximation, based on the definition of the Reynolds number as the ratio of the inertia and viscosity forces. The inertia force is represented by the momentum flux of the flowing mixture, whereas it is assumed that the viscosity forces are only present in the liquid phase, as there is always a boundary layer of liquid at the pipe wall and moreover $\eta_G \ll \eta_L$. Thus

$$\mathrm{Re}_{Dtp} = \frac{\alpha\rho_G u_G^2 + (1-\alpha)\rho_L u_L^2}{\eta_L \frac{u_L}{D}}.$$

As $\rho_G \ll \rho_L$ this may further be simplified to

$$\mathrm{Re}_{Dtp} = \frac{(1-\alpha)\rho_L u_L^2}{\eta_L \frac{u_L}{D}}$$

or

$$\mathrm{Re}_{Dtp} = \frac{(1-\alpha)\rho_L u_L D}{\eta_L} \qquad [29]$$

4. DETERMINATION OF ϵ_i

After determining expressions for the turbulence viscosity ϵ and for the velocity gradient (du_{Lz}/dr) it is then possible to compute (ϵ_i/ν) from [26], provided all other variables are known from measurement data. In the absence of an analytical model the results thus obtained will be factored into an empirical correlation.

4.1 *Available experimental data*

From the vast amount of experimental data on two-phase flow pressure loss only those easily accesible to the author have been used. A total of 253 measurements on vertical upflow have been gathered by Wisman (1975) and Stoop (1975), from five sources, viz. Cise (1964), Malnes (1966), Niese (1968), Muscettola (1963) and Janssen & Kervinen (1964), after further screening intended to eliminate systematic errors in the experiments. This dataset was extended by the author with 22 measurements on air–water flow in a pipe of 0.1 m dia. carried out by Wisman (unpublished work). The six sources are tabled in figure 3, where some characteristic data are given. These data cover a wide range of relevant flow parameters, hence all flow patterns are represented as can be seen from the flow pattern map of Hewitt & Roberts (1969) in figure 4. The void fraction values from Cise, Malnes and Niese were computed from the measured frictional and total pressure drops, whereas for the steam–water experiments of Muscettola and Janssen & Kervinen the void fraction values were computed using the slip correlation of Bankoff–Jones. Detailed information on this topic is given in the individual reports. All data consists of values averaged over a cross-section.

4.2 *Correlation for ϵ_i*

The values of (ϵ_i/ν) obtained from [26] and [16] for $2r/D = 0.816$ were plotted vs different dimensionless numbers expected to be relevent to interaction induced turbulence. It turned out that (ϵ_i/ν) is dependent on the product of three dimensionless numbers, viz. the Reynolds number for two-phase flow, defined in [29], the velocity ratio s, defined as u_G/u_L and the quotient $\alpha/(1-\alpha)$. The product of these numbers yields the number $\alpha u_G \rho_L D/\eta_L$. Figure 5 shows the values for (ϵ_i/ν) plotted vs this number. The resulting points might be correlated by

$$\frac{\epsilon_i}{\nu} = 0.0029\,\alpha u_G \rho_L D/\eta_L\,. \qquad [30]$$

The accuracy of this equation was evaluated using the statistical deviations δ and σ as defined by [22]–[24]. This resulted in an M.D. $\bar{\delta} = 1.1$ per cent and an S.D. $\sigma = 46.3$ per cent. While it appeared likely that better approximation of the values (ϵ_i/ν) can be obtained by a relationship of the form

$$\frac{\epsilon_i}{\nu} = a + b\alpha u_G \rho_L D/\eta_L$$

in which a and b are constants, this correlation does not yield a smooth transition to single phase liquid flow, as it is clear that (ϵ_i/ν) should be zero in case of vanishing void fraction. Hence we tried to find a correlation of the form

$$\frac{\epsilon_i}{\nu} = \alpha(a + bu_G \rho_L D/\eta_L)$$

Figure 6 shows the values of $(\epsilon_i/\alpha\nu)$ plotted vs the number $u_G \rho_L D/\eta_L$. These values are approximated by the correlation

$$\frac{\epsilon_i}{\nu} = \alpha(100 + 0.0024 u_G \rho_L D/\eta_L)\,. \qquad [31]$$

Source	number of measurements	media	pressure (bar)	pipe diameter (mm)	void fraction (-)	symbol in plots
CISE (1964)	71	argon/water	11.6 - 22	25	0.20 - 0.90	Δ
MALNES (1966)	69	air/water	1.0 - 1.4	24.7	0 - 0.58	+
NIESE (1963)	41	air/water	≈ 1.1	114.3	0 - 0.33	○
MUSCETTOLA (1963)	52	steam/water	41 - 82	5.0/5.2/6.3/10.1	0.20 - 0.65	×
JANSSEN and KERVINEN (1964)	20	steam/water	42 - 98	24	0.55 - 0.90	*
WISMAN	22	air/water	1.1 - 1.4	100	0.25 - 0.75	□

Figure 3. Table of sources of experimental data.

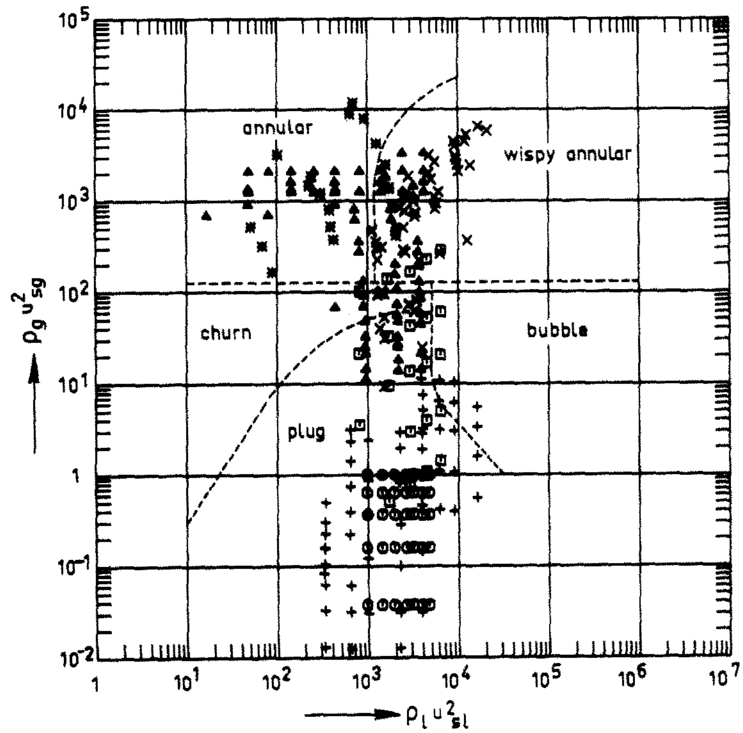

Figure 4. Flow pattern map.

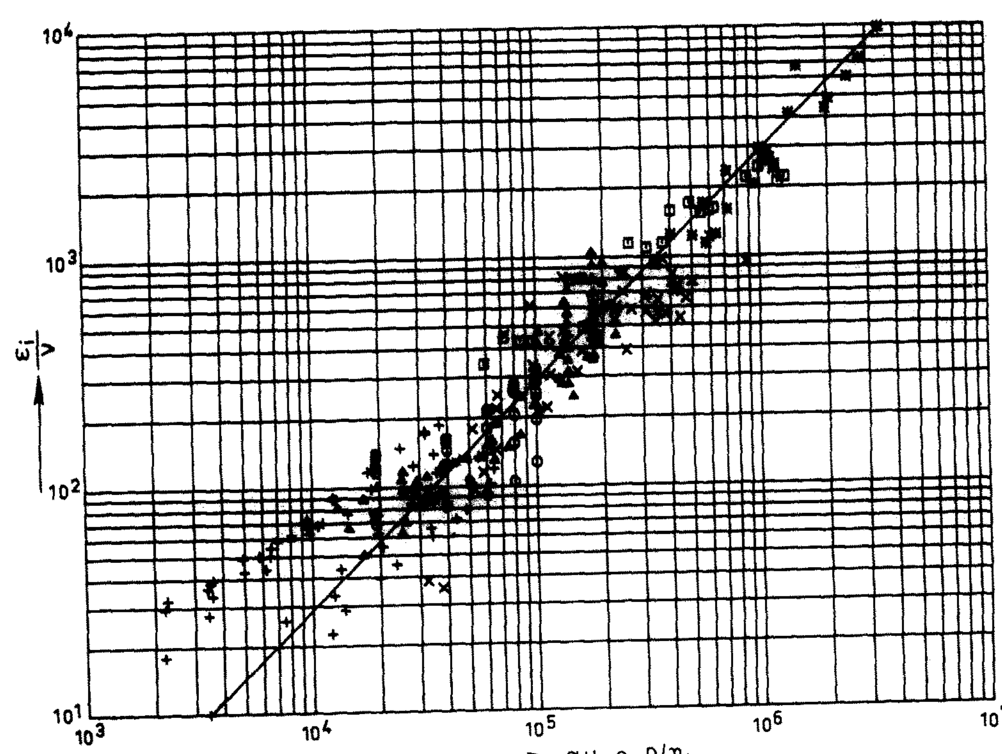

Figure 5. Computed (ϵ_i/ν) values as a function of the number $\alpha u_G \rho_L D/n_L$.

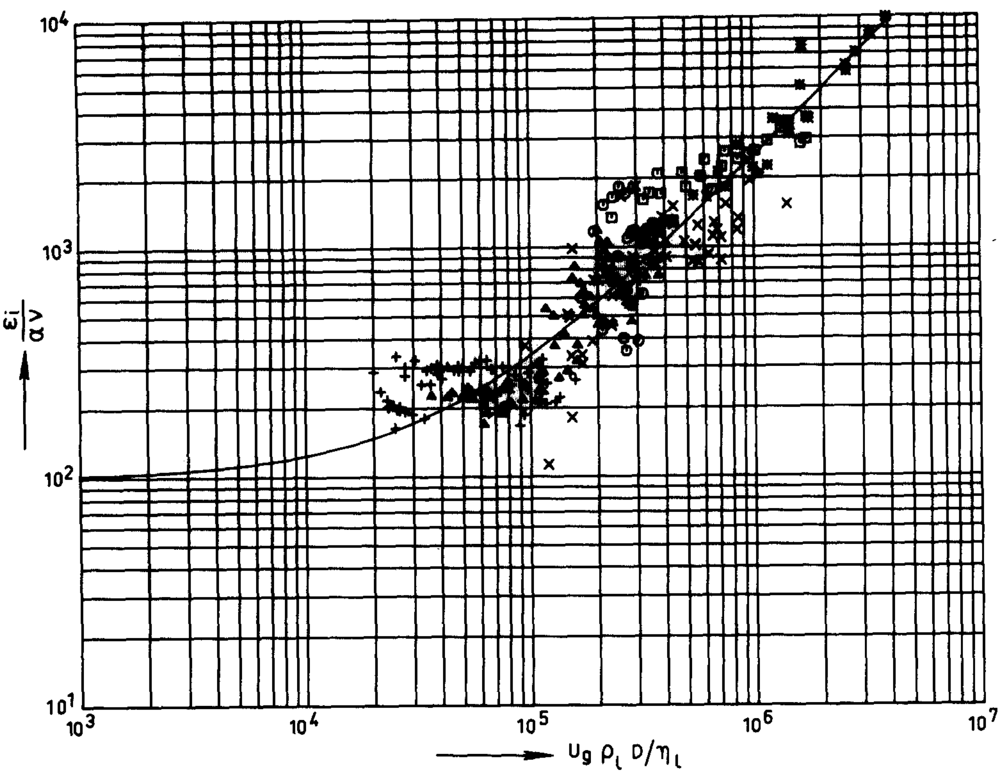

Figure 6. Computed $(\epsilon_i/\alpha\nu)$ values as a function of the number $u_G\rho_L D/\eta_L$.

As indicated by figure 6 the scatter around this correlation is significant, a fact reflected by the M.D. and S.D. values of $\bar{\delta} = 6$ per cent and $\sigma = 41$ per cent, respectively. However, introduction of the (ϵ_i/ν) values from [31] into the frictional pressure drop correlation [26] results in a rather high accuracy, as borne out by an M.D. $\bar{\delta} = 1.8$ per cent and an S.D. $\sigma = 22.3$ per cent.

The M.D. value of 6 per cent can be reduced to zero by optimizing the coefficients in [31], but this causes increased values for the deviations in correlation [26]. This is due to the fact that only part of the total frictional pressure drop is caused by phase interaction. Inaccuracies in the terms (ϵ_m/ν) and $du_{Lz}/dr)$ are in fact assigned to the term (ϵ_i/ν). In figure 7 the frictional pressure loss computed from [26], where (ϵ_i/ν) is given by [31], is plotted vs the measured pressure loss, showing good accuracy.

The relative magnitude of ϵ_i with respect to ϵ_m is plotted in figure 8 vs the quotient of the superficial velocities u_{sG}/u_{sL}, equivalent with $\alpha u_G/(1-\alpha)u_L$. From this figure it can be seen that $\epsilon_i \gg \epsilon_m$ in case of $u_{sG}/u_{sL} \gg 1$, which is to be expected since ϵ_i and ϵ_m are proportional to u_G and u_L, respectively.

Finally a remark on the applicability of the model to flows with high void fraction (i.e. wispy annular and annular flows) appears in order. The model is based on the assumption that the liquid phase is continuous and the viscosity forces are only present in this phase. It will hence be obvious that its applicability to flows with high void fractions should be considered highly questionable and the good correlation with experimental pressure drops found in this range a fortunate coincidence rather than a confirmation.

5. CONCLUSIONS

A correlation for the turbulence viscosity due to phase interaction in gas-liquid flow was developed, which shows satisfactory agreement with experimental data considering the margin of uncertainty introduced by the underlying assumptions. However, the present correlation

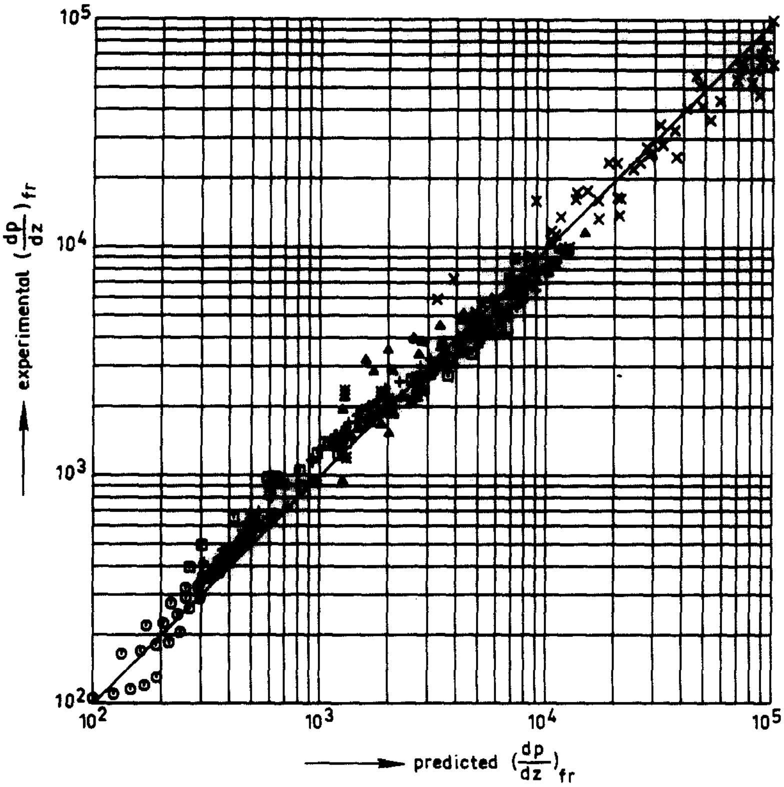

Figure 7. Comparison of experimental and predicted pressure loss.

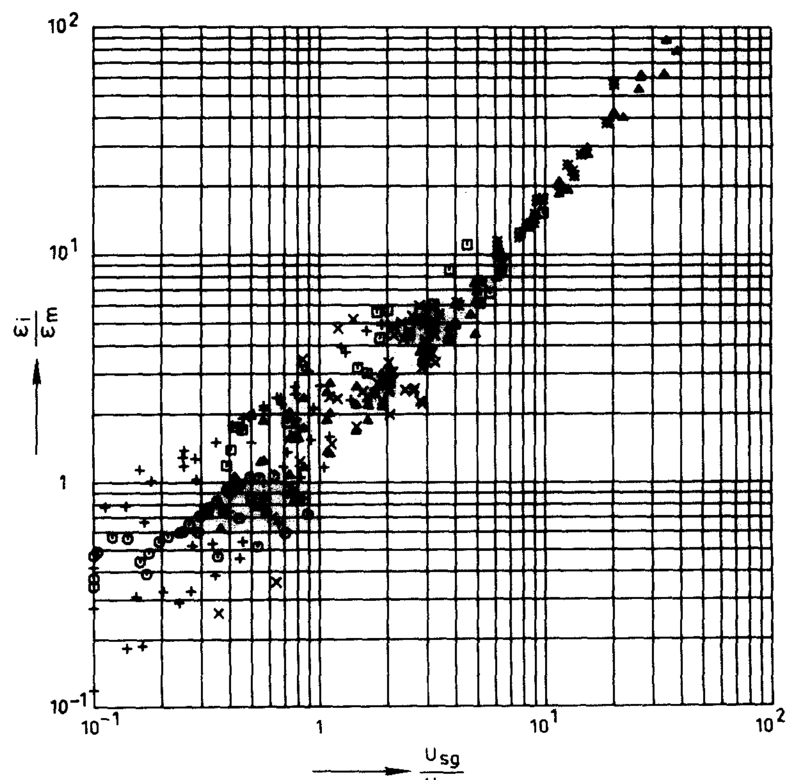

Figure 8. (ϵ_i/ϵ_m) values as a function of (u_{sG}/u_{sL}).

refers to values of (ϵ_t/ν) in the core region of the channel, whereas also local values in the wall region are needed to determine the second order viscosity terms in the momentum balances for two-dimensional computations. Such local values of the turbulence viscosity can only be found with the aid of measurements of velocity distributions and shear stress. One way to overcome this problem would be to assume a radial distribution function for the turbulence viscosity, such as the one proposed by Reichardt (1951). However, a discussion of this application is beyond the scope of the present note.

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